NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WARTIME REPORT

ORIGINALLY ISSUED
November 1945 as
Advance Restricted Report L5H30

STATISTICAL ANALYSIS OF THE CHARACTERISTICS OF REPEATED GUSTS IN TURBULENT AIR

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NACA

WASHINGTON

NACA ARR No. L5H30 [redacted]

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SUMMARY

Statistical methods have been applied to acceleration and airspeed data obtained with the XC-35 airplane during flights in turbulent air within convective clouds to determine some of the characteristics of repeated or closely spaced gusts pertinent to design problems. The results indicated that, in turbulent air within convective clouds, gusts tend to be contiguous and are seldom found isolated in space. In about two-thirds of the cases, successive gusts were opposite in direction and the first and the second gust seemed to have an equal chance of being the larger. The over-all average spacing between repeated gusts was in good agreement with twice the average gust-gradient distance of 10 chords used in the present design requirements.

For sets of two and three repeated gusts, average absolute gust intensities in the neighborhood of the design gust velocity of 30 feet per second were encountered. Within the scope of the data presented, sets of two and three repeated gusts with average absolute effective gust velocities of 25 feet per second and 23.5 feet per second, respectively, apperently will be encountered in turbulent air as often as single gusts of 30 feet per second.

INTRODUCTION

In the past, consideration of the intensity and frequency of occurrence of large single gusts has led to a reasonable set of design gust conditions. The design conditions so derived neglect the response of the airplane

[redacted]

to gusts previously encountered and are based on the assumption that, over a reasonable range of airplane characteristics, the motions of all conventional airplanes are similar. An airplane may, however, encounter a sequence of gusts in rapid succession with such spacings, directions, and intensities that the load impulses are placed in resonance either with the wing frequency or with the frequency of the pitching motions as determined by stability and control characteristics. It has been demonstrated in reference 1 that resonance with the wing frequency can lead to serious dynamic overstress of the wing structure. The existence of the problem of resonance with the frequency of pitching motions has been recognized in reference 2. If these problems are to be considered in connection with a particular airplane, the designer must have information concerning the probability that the sets of gusts having the spacings, directions, and intensities critical for the designed structure or stability characteristics will be encountered.

An opportunity to obtain such information is afforded by acceleration and airspeed data obtained with the XC-35 airplane (reference 3) during flights through turbulent air in convective clouds. Analysis of these data indicated that statistical procedures could be utilized to determine some of the more important properties of repeated gusts. These data, therefore, have been re-examined to obtain information concerning spacings, directions, and intensities of sets of gusts in a form amenable to statistical analysis.

Since critical sets of gusts are determined by individual airplane characteristics, no definite recommendations concerning design conditions for repeated gusts can be made. Hence this paper will merely indicate the methods of analysis and will present results for as many phases of the repeated-gust problem as is feasible in a form suitable for application to aerodynamic and structural design problems.

METHODS AND RESULTS

Basic Data

The data for the repeated-gust analysis were taken from time-history records of airspeed and acceleration obtained with the XC-35 airplane during flights through turbulent air within convective clouds. A typical acceleration time history is shown in figure 1. These records had already been evaluated for reference 3 and were in a convenient form for the present analysis. The time interval between successive gust peaks had been recorded, and effective gust velocities U_e had been computed by means of the sharp-edged-gust formula (reference 4)

$$\Delta n = \frac{\rho_o U_e V_e m}{2W/S}$$

where

Δn acceleration increment normal to chord of wing, g units

ρ_o mass density of air at sea level, slugs per cubic foot

U_e effective gust velocity, feet per second

V_e equivalent airspeed (the airspeed related to the true airspeed by the density ratio and equal to $V\sigma^{1/2}$), feet per second

V true airspeed, feet per second

σ relative density

m slope of lift curve, per radian

W weight of airplane, pounds

S wing area, square feet

It should be noted that the effective gust velocity defined by this formula is a fictitious quantity which is a measure, under average conditions, of the intensity of the true gust (reference 4) and permits the transfer of acceleration data obtained on one airplane to any other airplane. The term "gust peak" as used in the present paper is merely a convenient designation for an acceleration peak that has been modified by means of the sharp-edged-gust formula to eliminate primary airplane characteristics.

Since preliminary examination of the data showed that, throughout the range of gust intensities, up and down gusts appeared to have an equal probability of being encountered, only the absolute values of effective gust velocity $|U_e|$ (that is, the numerical values without regard to direction) have been considered.

The first step in the analysis was the determination of a spacing between gust peaks such that gusts with less than this spacing could reasonably be classed as repeated. Because of the manner in which the data had been evaluated, this maximum spacing could be expressed more conveniently as an increment of time than as an increment of distance. Examination of the data disclosed that the maximum time interval for a gust-gradient distance (distance from the beginning of a gust to the attainment of peak gust velocity) for all gusts encountered during the present investigation was 1.1 seconds. With the assumption that repeated gusts are essentially symmetrical so that the distance from the beginning of the gust to the attainment of peak gust velocity is the same as the distance from the gust peak to the end of the gust, twice the value of 1.1 seconds was selected as the maximum value. Thus, for the purposes of this report, a set of gusts is classed as repeated if the time interval between successive gust peaks is less than 2.2 seconds. In figure 1, for example, a and b form a set of two repeated gusts, c is isolated in space, and d to h form a set of five repeated gusts.

Method of Counting

Within any set of repeated gusts, there exists one or more smaller sets. The set of five repeated gusts in figure 1 contains five single gusts, four sets of two repeated gusts (de, ef, fg, and gh), three sets of three repeated gusts (def, efg, and fgh), and so forth. Accordingly, all the gusts (7800 gusts) were included in the analysis of the single gusts; all the gusts of the repeated-gust class (6800 gusts) were included in the analysis of two repeated gusts; all the gusts that occurred in sets of three or more repeated gusts (5200 gusts) were included in the analysis of three repeated gusts; and so on. The probability, based on values such as those just given, that a gust will occur in a set of N or more repeated gusts (N = 2, 3, 4, ...) is shown in figure 2. (Probability may be interpreted herein as the ratio of the number of gusts satisfying a given condition to the total number of gusts encountered. Thus, if the probability that a gust will exceed a given velocity is P, an average of one in every 1/P gusts will exceed that velocity.) Throughout this report, the symbol N will be used to indicate the number of gusts in a set of repeated gusts.

Statistical Procedures

The statistical methods employed permit the representation of the gust-velocity data in terms of three statistical parameters. The basic assumption is that the distributions of effective gust velocities which occur in sets containing different numbers of repeated gusts can be represented by Pearson Type III probability curves (reference 5). These curves form a three-parameter family; the parameters for a particular distribution are determined from the average absolute effective gust velocity $|U_e|_{av}$, the standard deviation σ, and the coefficient of skewness α_3 of the distribution. Specification of $|U_e|_{av}$, σ, and α_3 is therefore sufficient to describe the distribution completely.

Values of $|U_e|_{av}$, σ, and α_3 were computed for gusts occurring in sets of N or more repeated gusts and the results are presented in figure 3 together with what seem to be reasonable trend lines. In order to determine the validity of the basic assumption and the dependability of the trend lines, Pearson Type III curves (curve A determined from the three parameters of the actual data and curve B from the three parameters obtained from the trend lines) are compared with the actual data in figure 4 for sets of five repeated gusts. The results in figure 4 indicate that the parameters obtained from the trend lines in figure 3 may reasonably be used to represent the data in future calculations. The actual computation of the Type III curves is somewhat involved and depends on tables that are not in common use. In order that the gust-velocity data might be presented in convenient form, curves similar to curve B in figure 4 were drawn for all values of N and the results summarized in figure 5, which gives the probabilities of exceeding various values of U_e for each value of N.

Although the probability that the design gust velocity of 30 feet per second will be exceeded (fig. 5) is greatest in sets of about 20 repeated gusts, the fact that such sets do not occur so frequently as do single gusts and sets of fewer repeated gusts (fig. 2) requires that adjustment in the probability levels be made to reduce all sets to a common basis. This adjustment may be made by multiplying the probabilities of figures 2 and 5 at equal values of N to obtain the probability that a gust will exceed a given value and occur in a set of N or more repeated gusts. For example, the probability that a gust in a set of 10 or more repeated gusts will exceed 30 feet per second is 0.0022 (fig. 5) and the probability that a gust will occur in a set of 10 or more repeated gusts is 0.25 (fig. 2), so that the probability that a gust will exceed 30 feet per second and occur in a set of 10 or more repeated gusts is 0.00055. In figure 6 these probabilities are shown for gusts exceeding 25, 30, and 35 feet per second.

Spacing

The values of spacing D between successive gust peaks were obtained from the time histories by use of the following expression:

$$D = \frac{V \, \Delta t}{9.23}$$

where 9.23 feet is the length of the mean aerodynamic chord for the XC-35 airplane and

D spacing, chords

Δt increment of time between successive gust peaks, seconds

V true airspeed, feet per second

Values of D were averaged for gusts occurring in sets of N or more repeated gusts, and the results are presented in figure 7 in which the trend in the average spacing D_{av} between sucessive gusts with increasing N is shown.

Sets of Two and Three Repeated Gusts

Since probability is considered herein as the ratio of the number of gusts that satisfy a given condition to the number of gusts encountered, there is no basis for comparing probabilities for different values of N unless the total number of repeated gusts is known for each value of N. Accordingly, in order that sets of two and three repeated gusts might be compared, the number of occurrences of each has been determined.

Since sets of three repeated gusts contain two sets of two repeated gusts, sets of four contain three sets of two repeated gusts, and so on, it follows that if M_j is the number of gusts which occur in sets of exactly j repeated gusts ($j = 2, 3, 4, \ldots$), then

$$\sum_j M_j \frac{j - 1}{j}$$

represents the total number of sets of two repeated gusts that will be encountered. Similarly,

$$\sum_j M_j \frac{j - 2}{j}$$

represents the total number of sets of three repeated gusts that will be encountered. Relative values of M_j were taken from figure 2 by subtracting successive values of probability. Substitution of these values into the foregoing expressions showed that the total number of sets of two and three repeated gusts is 65 percent and 47 percent, respectively, of the total number of gusts encountered. With the 7800 gusts of the present investigation, there are about 5100 sets of two repeated gusts and about 3700 sets of three repeated gusts. The average absolute effective gust velocity $|U_e|_{av}$ was computed for each of these sets and the results are presented in figure 8, which shows the number of sets of two and three repeated gusts with $|U_e|_{av}$ exceeding a given value, expressed in percent of the total number of gusts encountered. For comparison, the distribution of all gusts encountered (that is, the single-gust condition) is also shown in figure 8.

The envelopes of intensities of gusts that make up the sets of two and three repeated gusts of large average absolute intensity $\left(|U_e|_{av} \geqq 15 \text{ fps}\right)$ are shown in figures 9 and 10, respectively. Figure 9 shows, for example, that within the scope of these data, a set of two repeated gusts with $|U_e|_{av} = 20$ feet per second might reasonably be expected to contain gusts with velocities from 8 feet per second to 32 feet per second.

The envelope of spacings for sets of two repeated gusts of large average absolute intensity $\left(|U_e|_{av} \geqq 15 \text{ fps}\right)$ is shown in figure 11. For example, for sets of two repeated gusts with $|U_e|_{av} = 20$ feet per second, the spacing between the two gusts of the set might reasonably be expected to be between about 9 and 58 chords. For sets of three repeated gusts, the average of the two spacings of the set has been taken as a reasonable measure of spacing. In figure 12, the limit of these average spacings is shown for sets with $|U_e|_{av} \geqq 15$ feet per second.

DISCUSSION

Inspection of figure 6 indicates that if gust intensity were the sole criterion for the determination of critical sets of repeated gusts for design purposes, single gusts and sets containing a small number of repeated gusts would be more important than sets containing a large number of repeated gusts. As has already been pointed out, however, directions and spacings must also be considered.

The over-all average spacing between repeated gusts was about 25 chords (fig. 7). One-half of this value is in good agreement with the average gust-gradient distance of 10 chords upon which the present design gust requirements are based. For gusts of large intensity (figs. 11 and 12) the average spacing was about 20 chords or twice the design gust-gradient distance. The trend in average spacings between gusts shown in figure 7 indicates that spacings between gusts tend to decrease as the number of gusts in a set of repeated gusts increases.

Obviously a detailed study could be made of the characteristics of sets of any number of repeated gusts.

Since information concerning sets containing a small number of repeated gusts would be expected to be of most value in structural design problems, however, only single gusts and sets of two and three repeated gusts have been analyzed in detail.

Single Gusts

Intensity.- The distribution of all gust velocities encountered during the investigation with the XC-35 airplane has been presented in figure 8. The probability that the airplane will exceed the design gust velocity of 30 feet per second is approximately 0.001; that is, an average of one gust in about 1000 will have a velocity greater than 30 feet per second. The number of gusts having a velocity greater than a given value that will be encountered by an airplane in turbulent air within convective clouds may be calculated by use of figure 8 and a reasonable estimate of the number of gusts per mile of turbulent air.

Approximately 17 percent of all the gusts encountered were isolated in space and none of these isolated gusts had a velocity greater than 15 feet per second. Thus, in turbulent air, gusts tend to be contiguous and are seldom found isolated in space.

Direction.- Up and down gusts appear to have an equal chance of being encountered.

Sets of Two Repeated Gusts

Intensity.- Examination of the data indicated that, in sets of two repeated gusts, the first and the second gusts have an equal chance of being the larger. The distribution of average absolute gust intensities $|U_e|_{av}$ for sets of two repeated gusts is shown in figure 8. As in the case of single gusts, this figure together with an estimate of gusts per mile of turbulent air permits the calculation of the number of sets of two repeated gusts with $|U_e|_{av}$ greater than a given value that will be encountered by an airplane in turbulent air.

Values of $|U_e|_{av}$ as high as 31 feet per second were encountered in sets of two repeated gusts. Figure 8 indicates that a set with $|U_e|_{av}$ greater than 25 feet per second will occur as often as a single gust of intensity greater than 30 feet per second.

The range of gust velocities that make up the sets of large average absolute intensity (fig. 9) indicates that, within the scope of the present data, for $|U_e|_{av}$ = 25 feet per second, a range of gust velocities from 15 to 35 feet per second can be assumed in design problems. Because of the small quantity of data obtained at the higher values of $|U_e|_{av}$, precise estimates of the range of gust velocities cannot be made. The envelope in figure 9 represents an estimate that could be improved if more data were available.

Direction.- As might be expected, an up or a down gust seems to have an equal chance of being the first encountered. In about two-thirds of the cases, however, the successive gusts are opposite in direction.

Spacing.- The average spacing between two repeated gusts was about 25 chords with a range of 5 to 100 chords. The range of spacings in sets of large intensity (fig. 11) indicates that, within the scope of the data, for $|U_e|_{av}$ = 25 feet per second, spacings of 15 to 32 chords can be assumed in design problems.

Sets of Three Repeated Gusts

Intensity.- The distribution of average absolute gust intensities $|U_e|_{av}$ for sets of three repeated gusts is shown in figure 8. As in the case of single gusts and sets of two repeated gusts, an estimate can be made from this figure of the number of sets of three repeated gusts with $|U_e|_{av}$ greater than a given value that will be encountered by an airplane in turbulent air.

Values of $|U_e|_{av}$ as high as 27 feet per second were encountered in sets of three repeated gusts. Figure 8 indicates that a set with $|U_e|_{av}$ greater then 23.5 feet per second will occur as often as will a single gust of intensity greater than 30 feet per second.

The range of gust velocities that make up the sets of large average absolute intensity (fig. 10) indicates that, within the scope of the present data, for $|U_e|_{av}$ = 23.5 feet per second, a range 10 to 36 feet per second can be assumed in design problems.

Direction.- No predominant combination of up and down gusts exists for sets of three repeated gusts. All combinations seem to have an equal chance of being encountered.

Spacing.- Spacing in a set of three repeated gusts has been characterized by the average of the two spacings between the three gusts in the set. The range of these average spacings for sets of large intensity (fig. 12) indicates that, within the scope of these data, for $|U_e|_{av}$ = 23.5 feet per second, average spacings from 12 to 32 chords can be assumed in design problems.

Although sets of more than three repeated gusts have not been analyzed in detail, recommendations concerning these sets apparently should be similar to those advanced for sets of two and three repeated gusts.

Remarks on Applications

In applying the foregoing results to design problems, a suitable forcing function must be selected to represent a given set of repeated gusts. The form of this forcing function may be determined to a first approximation from the shapes of the acceleration peaks involved. A wide variety of peak shapes were encountered but most of the peaks were approximately sinusoidal. It has already been noted that, for repeated gusts of large intensity, the average spacing between peaks was twice the average gust-gradient distance; that is, for these gusts little smooth air is encountered between successive gusts. These facts, together with the assumption of gust symmetry, suggest that the forcing function should be sinusoidal with the distance between successive zeros equal to the average spacing between the peaks and the amplitudes determined by the intensities of the individual gusts.

The results of figures 8 to 12 have been summarized in table I, which presents the characteristics of sets of two and three repeated gusts with average absolute intensities that will be encountered as frequently as single gusts with absolute intensities of 20, 25, 30, and 35 feet per second. The amplitude and period of the forcing function may be selected on the basis of this table. Inasmuch as any of the gusts in a set of repeated gusts appears to have an equal probability of being the larger, average conditions can be approximated if all the gusts are assumed to have the same velocity. The amplitude of the forcing function may therefore be determined from the average absolute gust velocity selected from table I. The impressed frequencies may be computed from the ranges of spacings in table I for comparison with the wing or pitching-motion frequencies.

CONCLUSIONS

Statistical methods have been applied to acceleration and airspeed data obtained with the XC-35 airplane during flights in turbulent air in convective clouds to determine some of the characteristics of repeated or closely spaced gusts pertinent to design problems. The following conclusions were indicated:

1. In turbulent air, within convective clouds, gusts tend to be contiguous and are seldom found isolated in space. All gusts of intensity greater than about 15 feet per second encountered during this investigation were in the repeated-gust class.

2. For sets of two repeated gusts, in about two-thirds of the sets, successive gusts were found to be opposite in direction and, as might be expected, either an up or a down gust had an equal chance of being the first encountered. Both the first and second gusts have an equal chance of being the larger.

3. The distribution of gust intensities that occur in sets containing different numbers of repeated gusts can be represented adequately by Pearson Type III probability curves.

4. Based on the consideration of gust intensity alone, single gusts and sets of two and three repeated gusts are more important than sets containing a large number of repeated gusts, since the probability of encountering gusts of large intensity in these sets is higher than in sets containing larger numbers of repeated gusts.

5. The over-all average spacing between sets of two repeated gusts is about 25 chords. The spacings between gusts tend to decrease as the number of repeated gusts in a set increases.

6. For sets of two and three repeated gusts, average absolute gust intensities in the neighborhood of the design gust velocity of 30 feet per second were encountered. Within the scope of present data, sets of two and three repeated gusts with average absolute effective gust velocities of 25 feet per second and 23.5 feet per second, respectively, apparently will be encountered in turbulent air as often as single gusts of intensity greater than 30 feet per second.

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REFERENCES

1. Pierce, Harold B.: Dynamic Stress Calculations for Two Airplanes in Various Gusts. NACA ARR, Sept. 1941.

2. Bryant, L. W., and Jones, I. M. W.: Stressing of Aeroplane Wings due to Symmetrical Gusts. R. & M. No. 1690, British A.R.C., 1936.

3. Moskovitz, A. I.: XC-35 Gust Research Project Bulletin No. 8 - Analysis of Gust Measurements. NACA RB No. L4D22, 1944.

4. Rhode, Richard V.: Gust Loads on Airplanes. SAE Jour., vol. 40, no. 3, March 1937, pp. 81-88.

5. Elderton, W. Palin: Frequency Curves and Correlation. Cambridge Univ. Press, 1938, pp. 92-95.

TABLE I

COMPARISON OF THE PERTINENT CHARACTERISTICS OF SINGLE GUSTS AND SETS OF TWO AND THREE REPEATED GUSTS OCCURRING WITH EQUAL FREQUENCY

Single gust	Set of two repeated gusts				Set of three repeated gusts			
Effective gust velocity (fps)	Effective gust velocity (fps)		Spacing (chords)		Effective gust velocity (fps)		Spacing (chords)	
$\|U_e\|$	$\|U_e\|_{av}$	Range	D_{av}	Range	$\|U_e\|_{av}$	Range	D_{av}	Range
20	17	5 to 29	28	5 to 64	15.5	7 to 31.5	26	8 to 43
25	21	9 to 33	28	12 to 54	19.5	8 to 33.5	24	8 to 41
30	25	15 to 35	22	15 to 32	23.5	9.5 to 36.0	20	12 to 32
35	29.5	21.5 to 37.5	22	15 to 30	27.5	10.5 to 38.5	20	15 to 25

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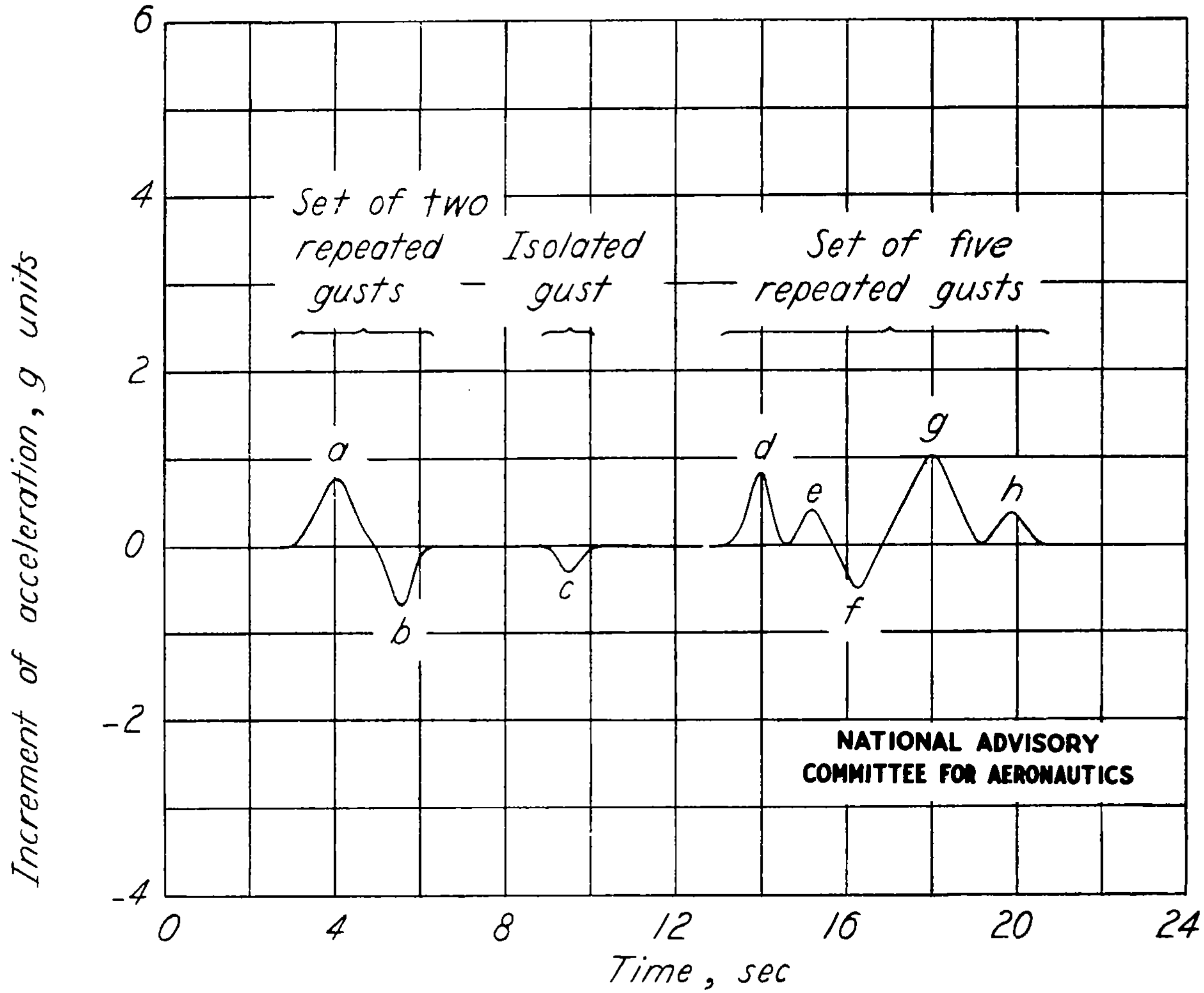

Figure 1.- Typical time history of acceleration.

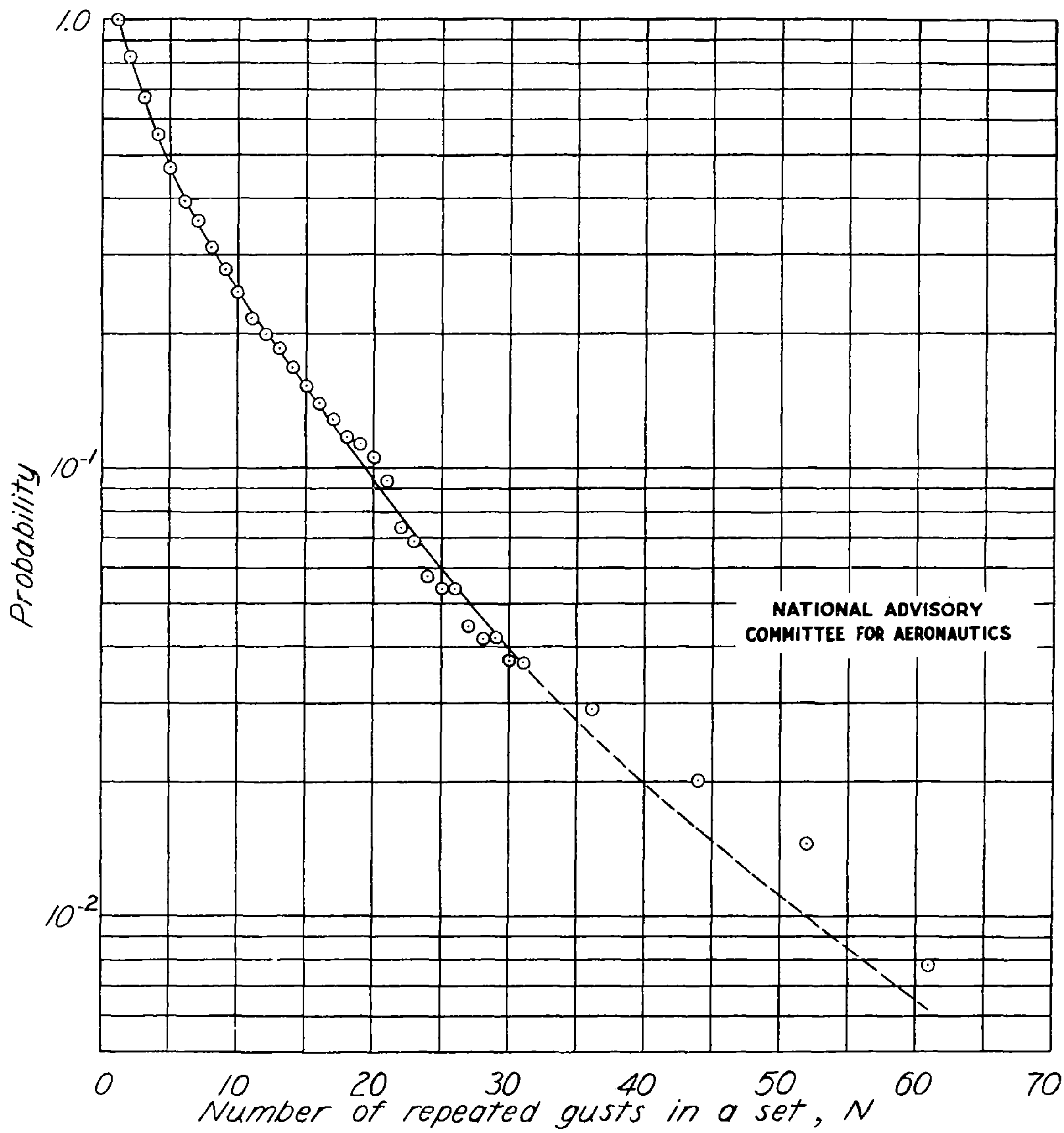

Figure 2.- Probability that a gust will occur in sets of N or more repeated gusts.

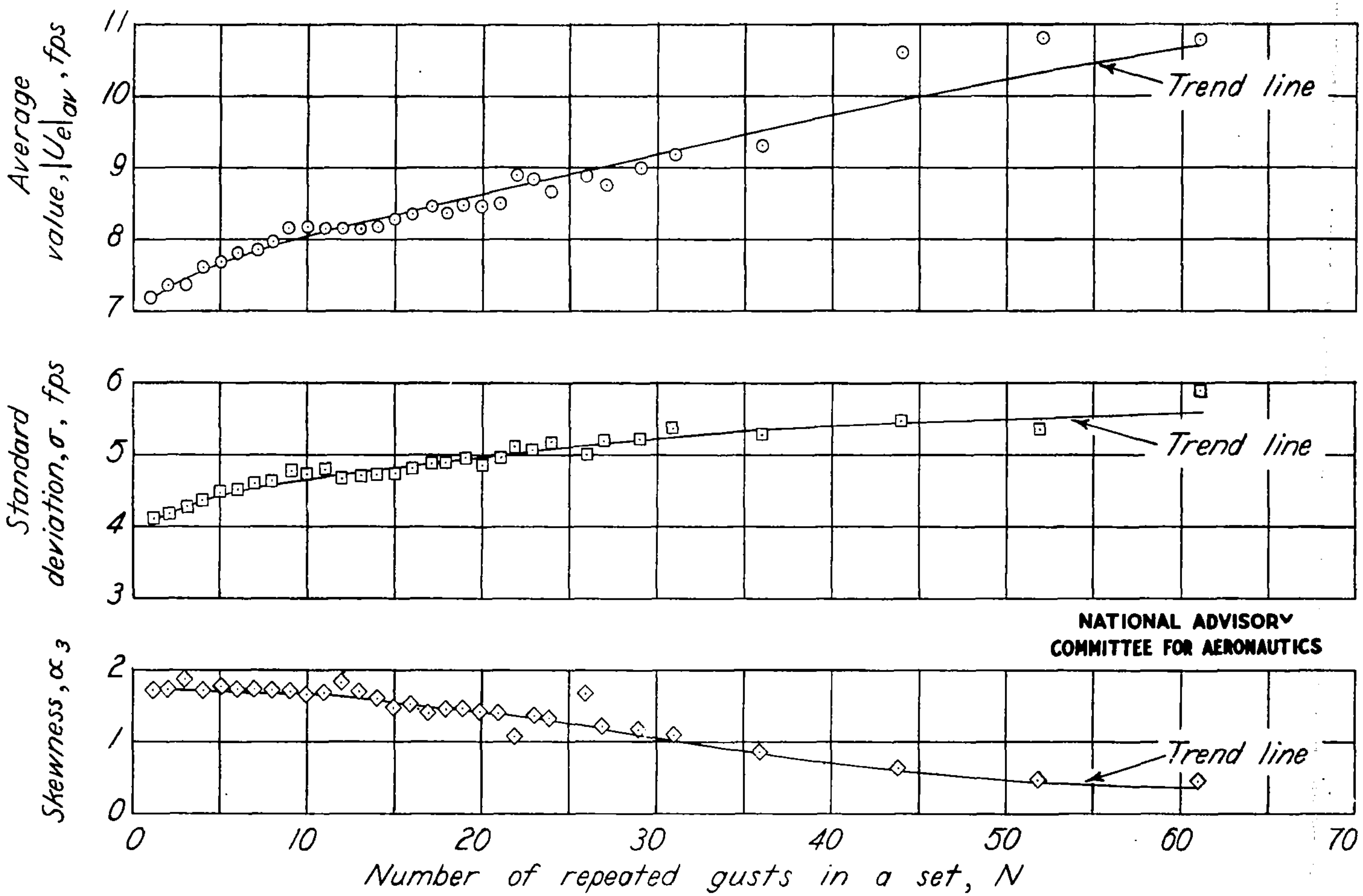

Figure 3.- Variations in the parameters of the distributions of effective gust velocity for N or more repeated gusts in a set.

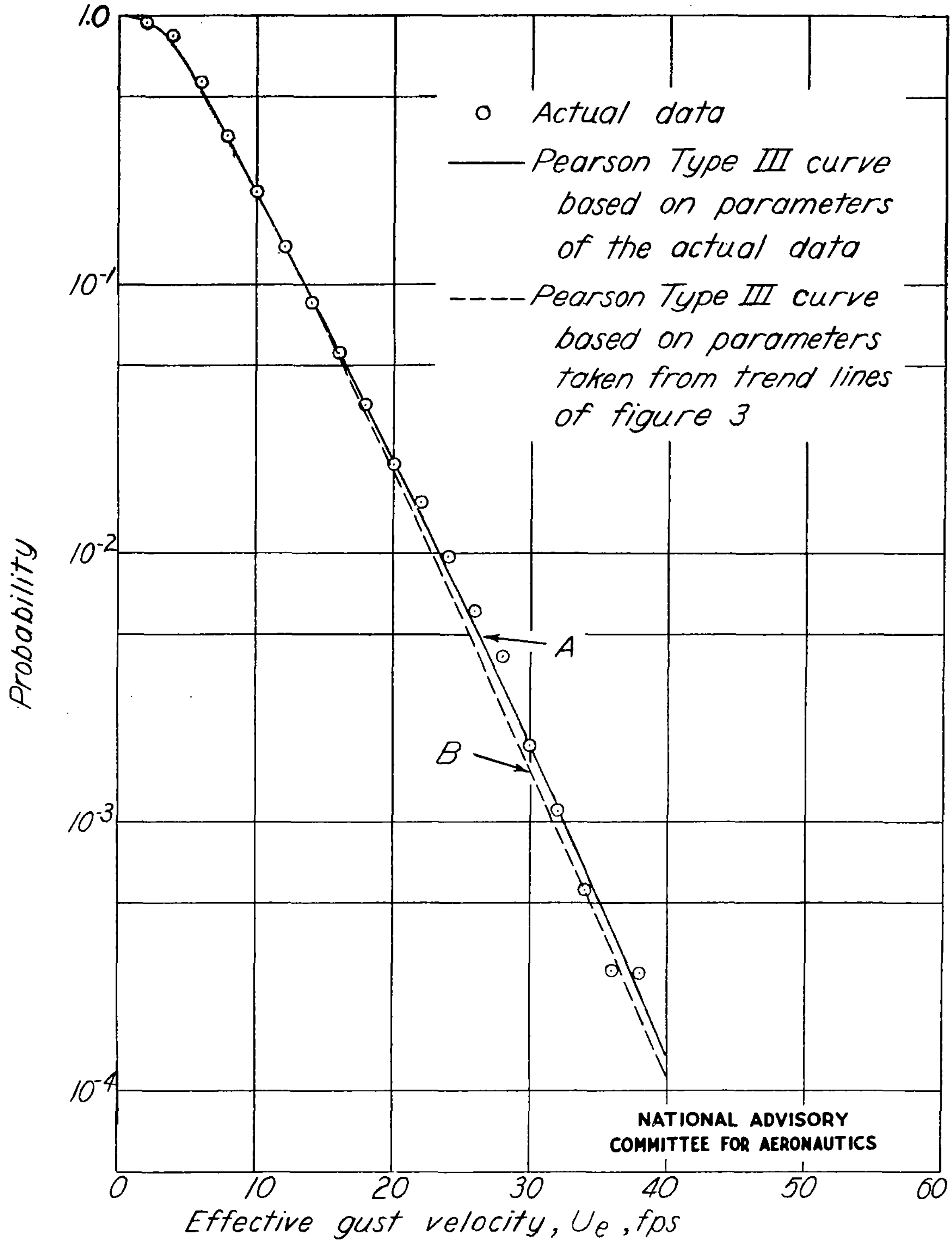

Figure 4 - Comparison of actual data with Pearson Type III probability curves for sets of five repeated gusts.

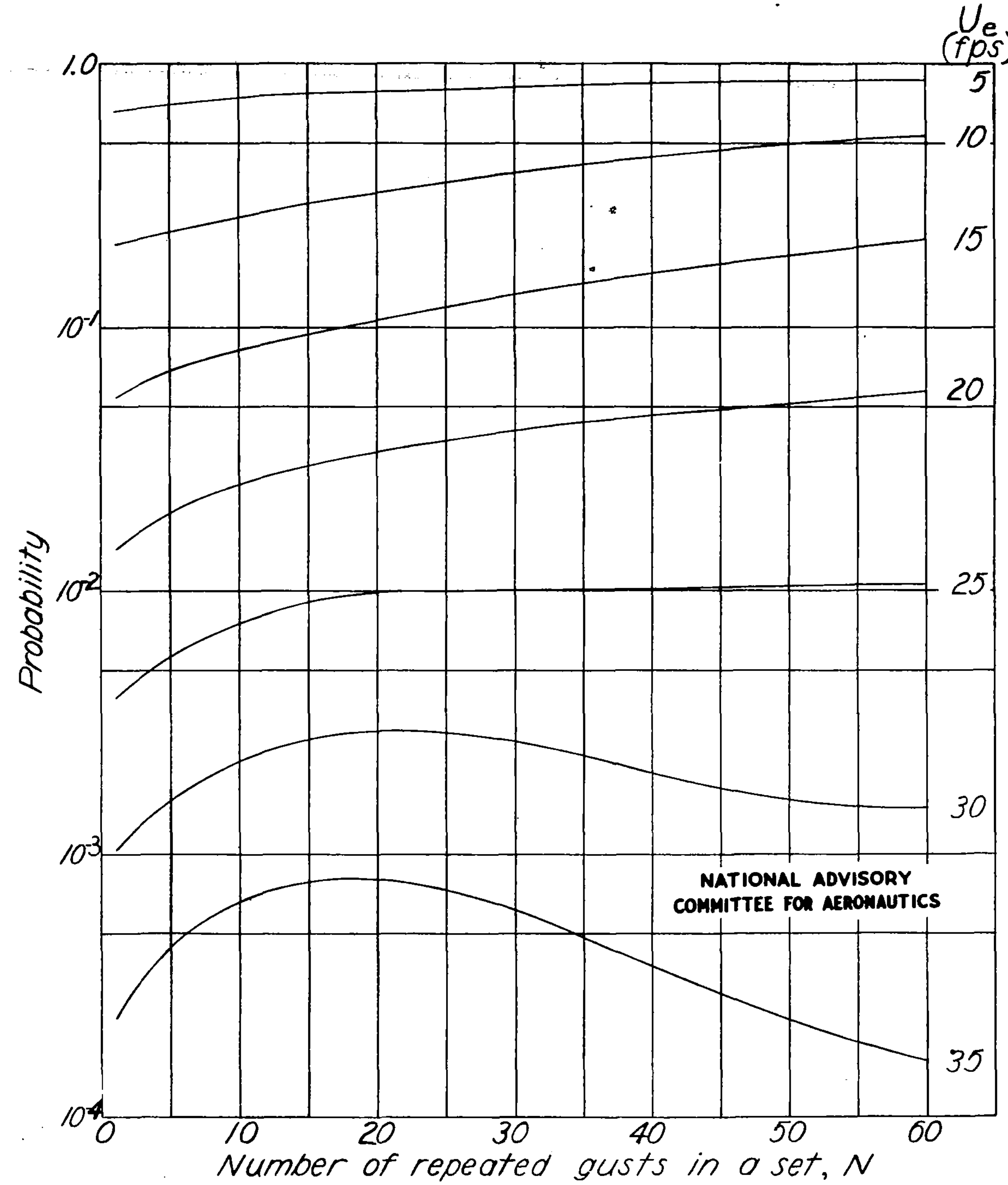

Figure 5.- Probability of exceeding a given value of effective gust velocity in a set of N or more repeated gusts.

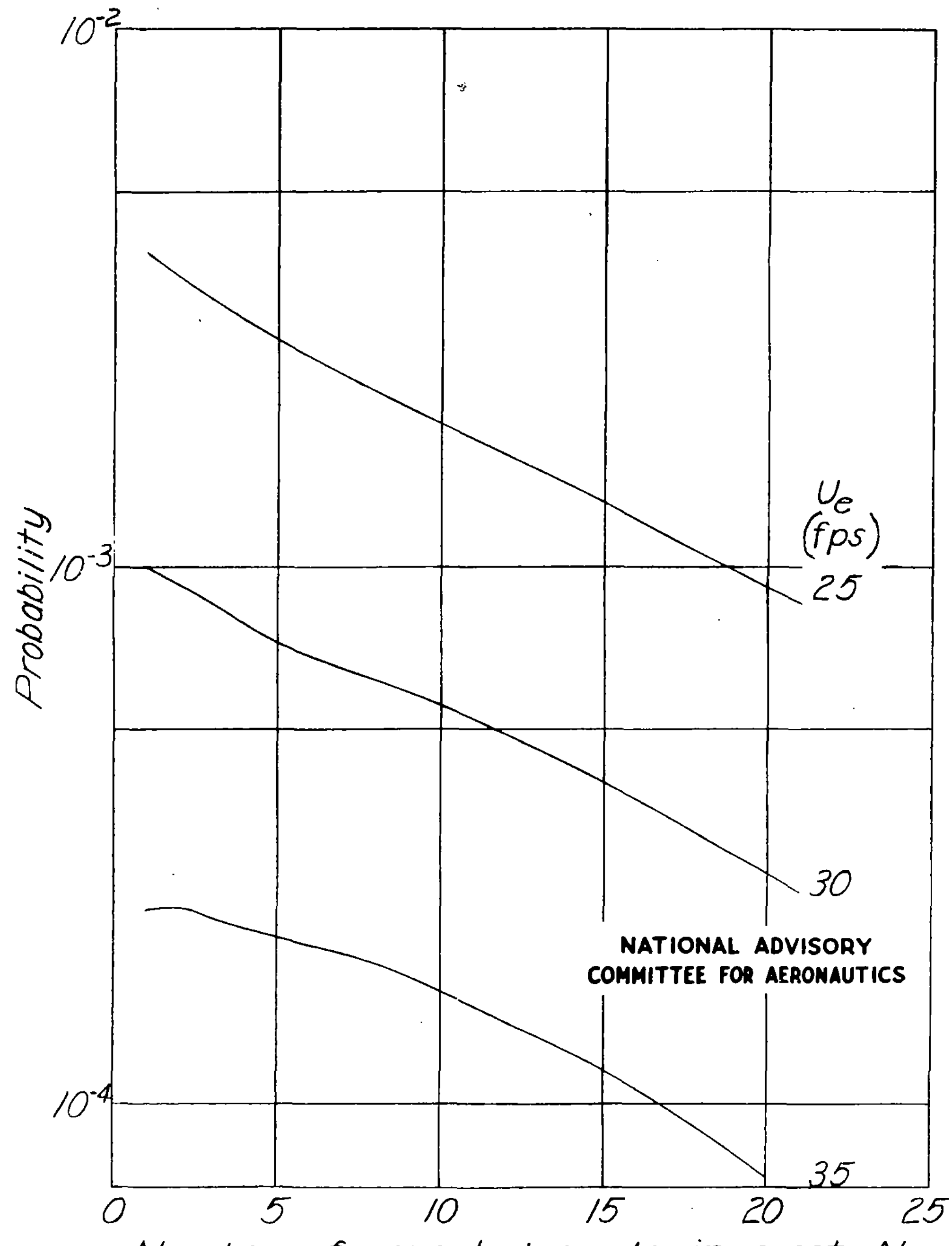

Figure 6.- Probability that a gust will exceed a given value of effective gust velocity and occur in a set of N or more repeated gusts

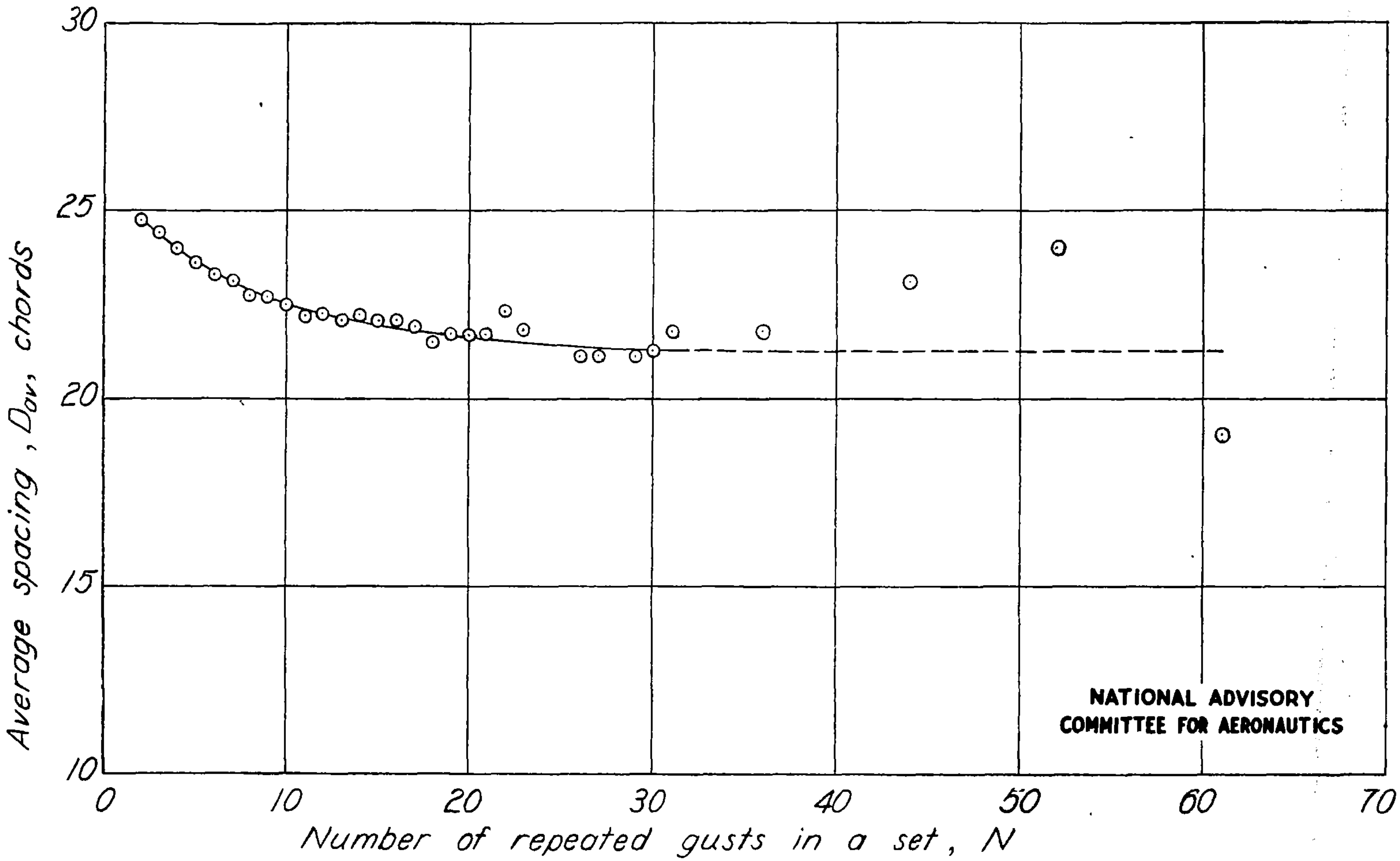

Figure 7.- Trend in the average spacing between successive gusts in sets of N or more repeated gusts.

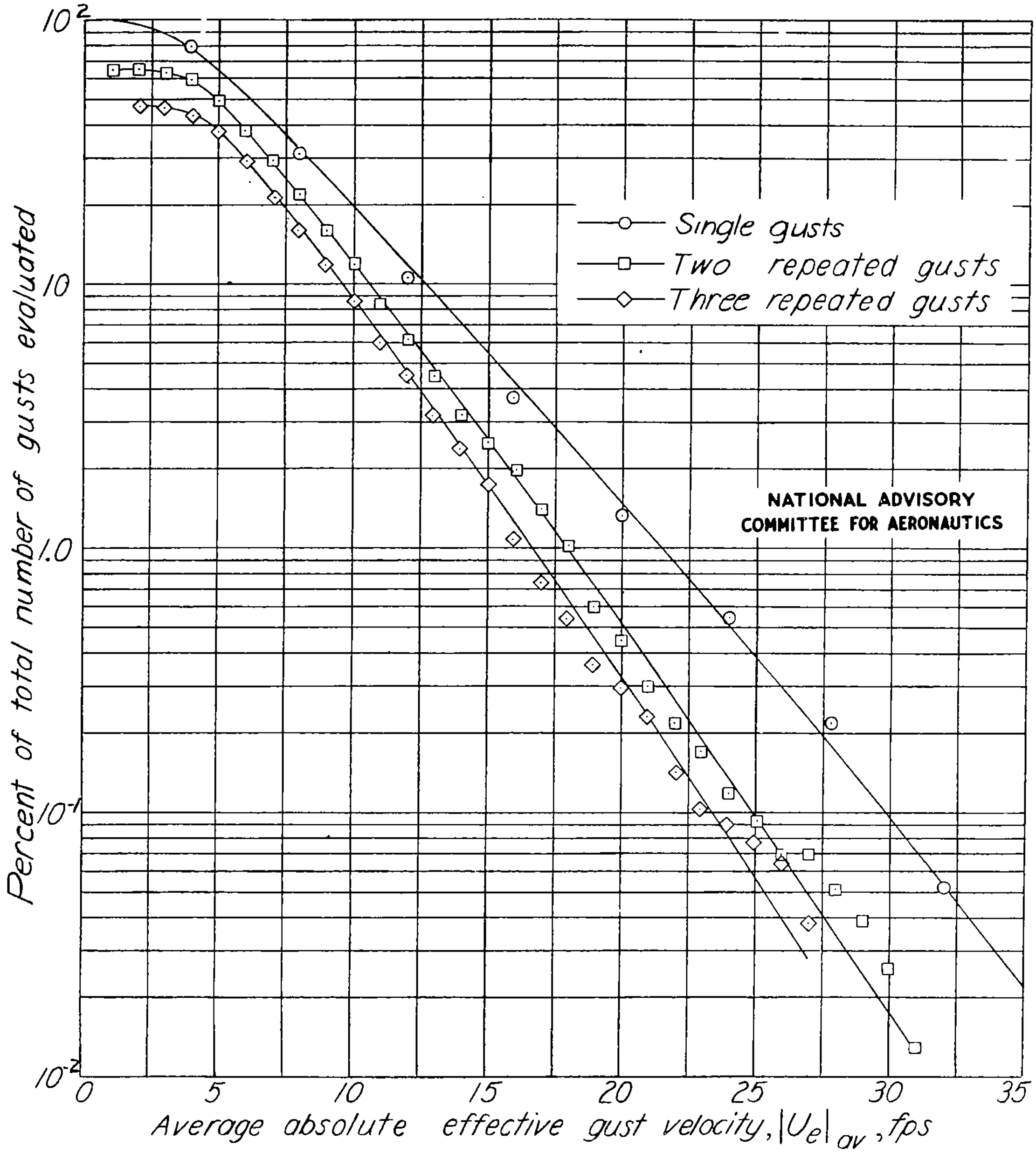

Figure 8.-Number of average absolute effective gust velocities greater than a given value for single gusts and sets of two and three repeated gusts, expressed in percent of total number of gusts evaluated.

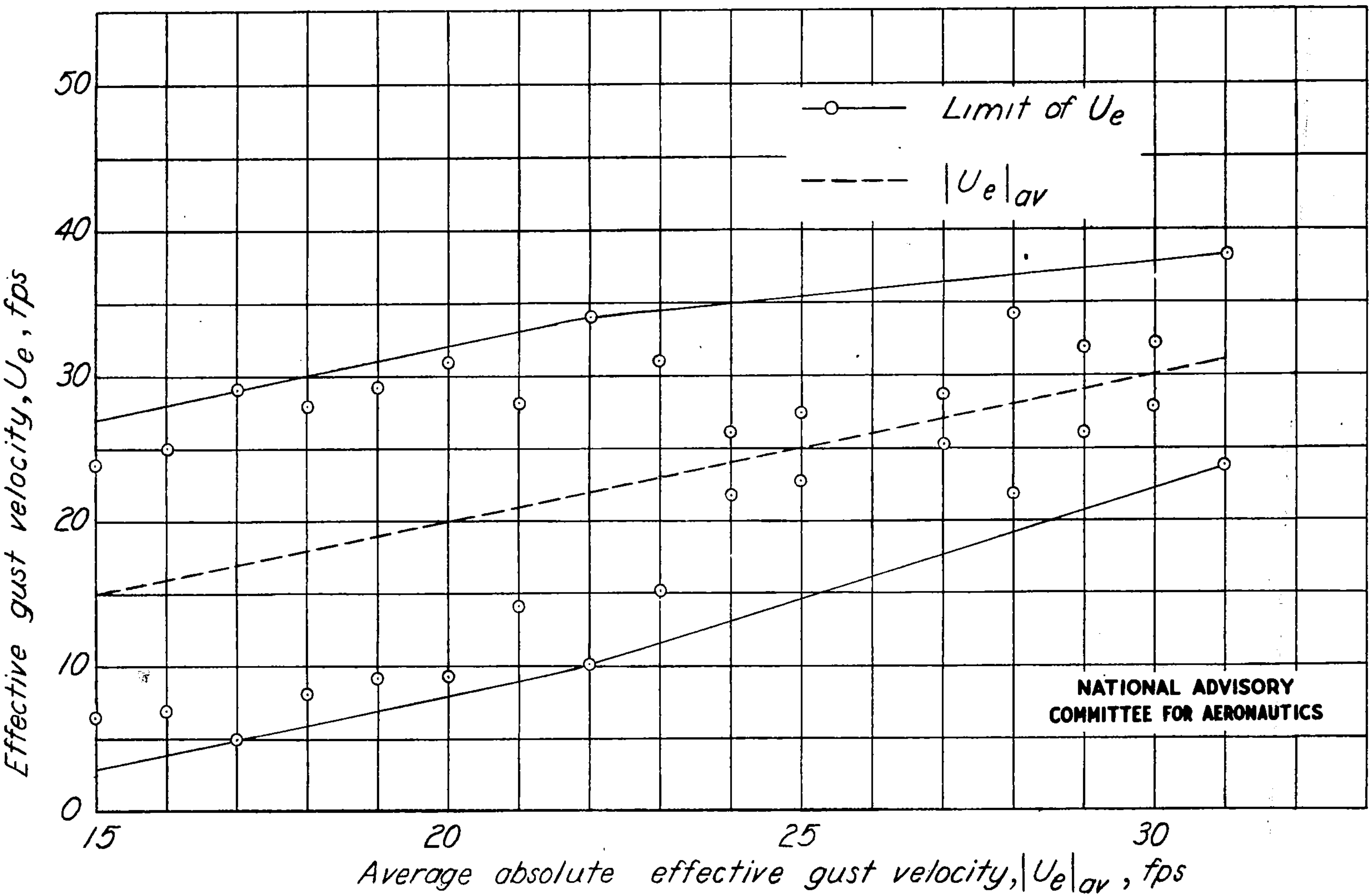

Figure 9.- Limit of effective gust velocities for sets of two repeated gusts.

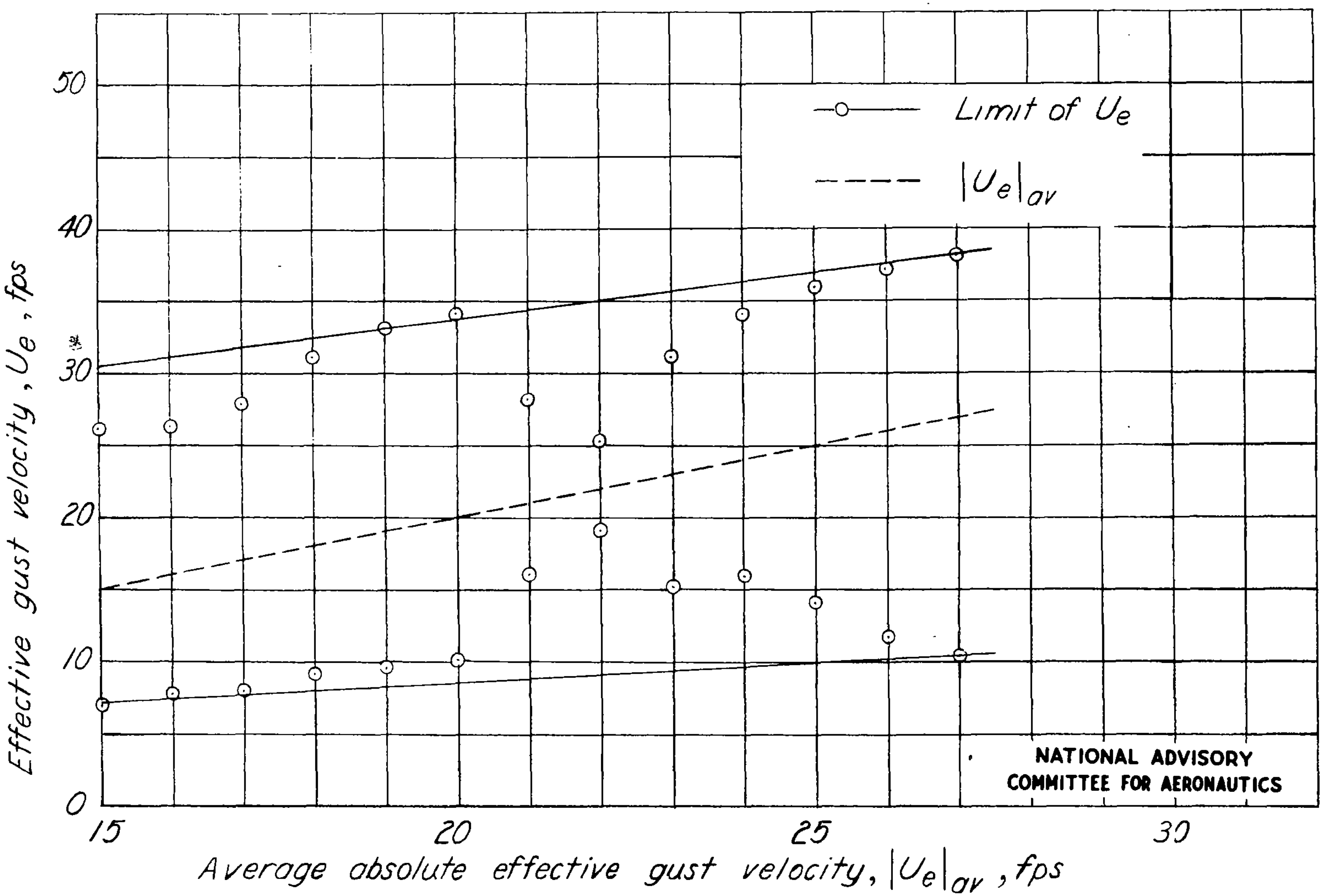

Figure 10.– Limit of effective gust velocities for sets of three repeated gusts

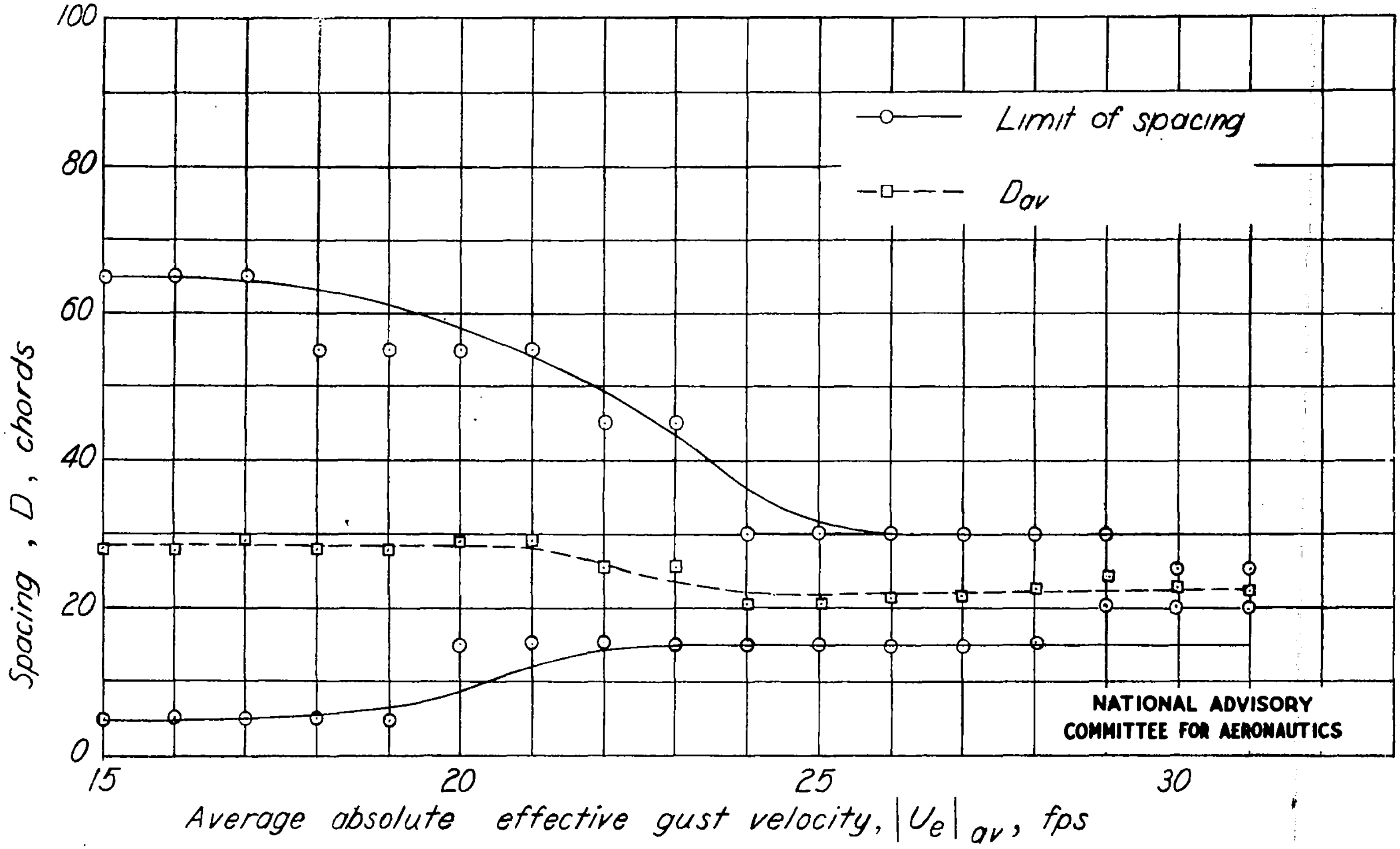

Figure 11.– Limit of spacings between two repeated gusts.

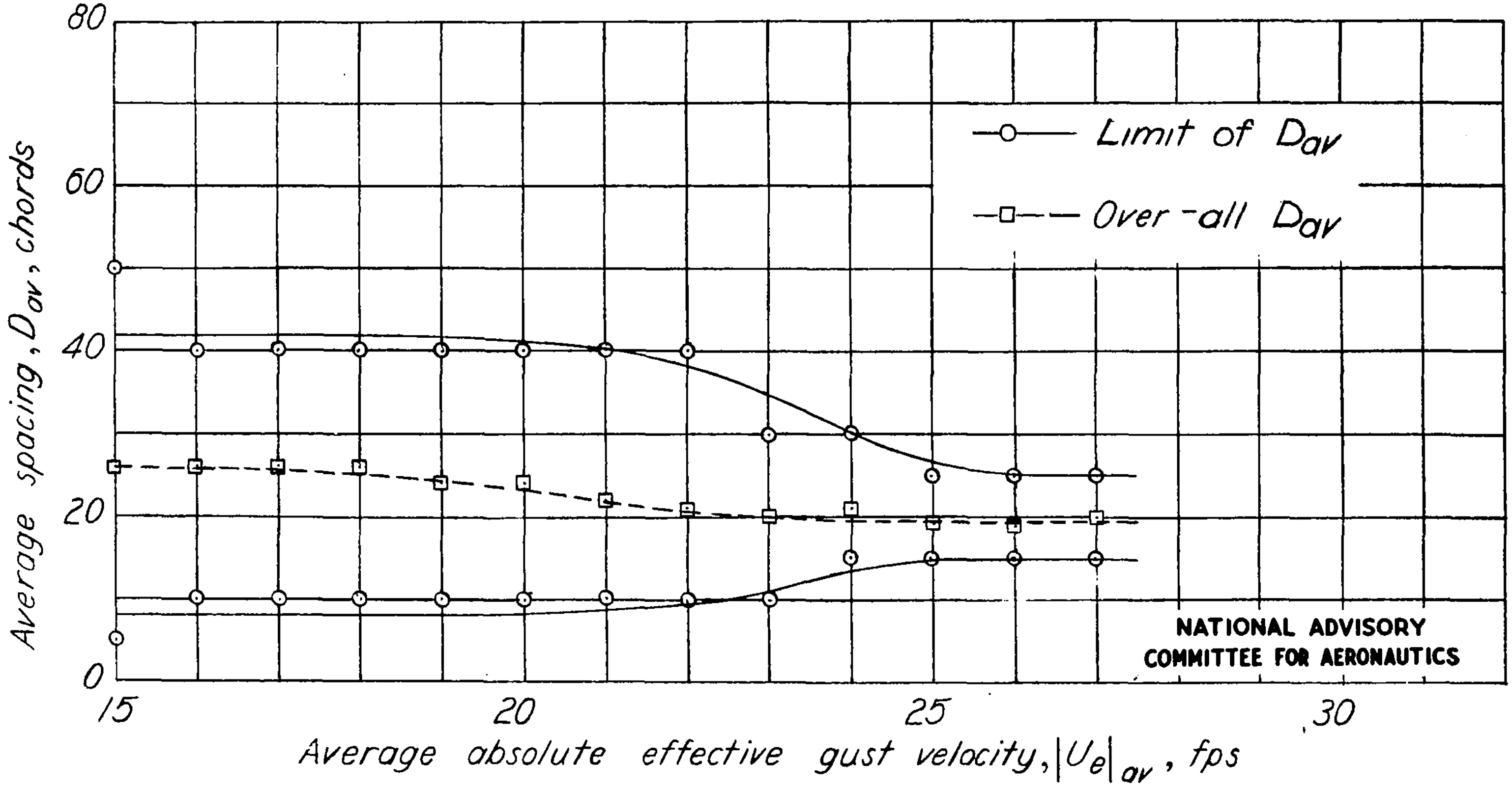

Figure 12.– Limit of average spacings between three repeated gusts.